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Quantum Fluctuations in a Mesoscopic Inductance Coupling Circuit

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Starting from the equation of motion of a mesoscopic inductance coupling circuit, the quantum fluctuations of charge and current in the circuit are investigated in both the eigenstates of the system and the squeezed vacuum state. The results show that there exist quantum fluctuations of the charge and current in both cases, and the fluctuations in each component circuit are connected.

1. INTRODUCTION

With the rapid development of nanoelectronics and nanometer-scale technology [1, 2] the electronic device community has been witnessing a strong trend toward atomic scale dimensions in the miniaturization of integrated circuits and components [3]. Quantum effects in electronic devices and circuits should be taken into account when the transport dimension reaches a characteristic dimension, namely, when the charge-carrier inelastic coherence length approaches the Fermi wavelength. In 1973, Louisell discussed the quantum effects in an *LC* circuit and gave its quantum noise in the vacuum state [4]. Recently, quantum effects in a mesoscopic capacitance coupling circuit were studied by Chen *et al.* [5] and Yu and Liu [6]. We investigated quantum effects in the charge and current of mesoscopic *LC* and *RLC* circuits in the squeezed vacuum state [7, 8]. As the vacuum state can be regarded as a special case of the squeezed vacuum state when the squeezing parameter is zero, it is more general and significant to study quantum effects

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in the charge and current in squeezed states of a mesoscopic circuit. In this paper, we study the quantum fluctuations of charge and current in eigenstates of the system and in the squeezed vacuum state of a nondissipative mesoscopic inductance coupling circuit.

2. THE QUANTIZATION OF A MESOSCOPIC INDUCTANCE COUPLING CIRCUIT AND ITS EIGENSTATES

In a nondissipative circuit with two LC circuits coupled by an inductance, according to Kirchhoff's law, the classical equations of motion are

$$\begin{aligned} L_1 \frac{d^2 q_1}{dt^2} + \frac{q_1}{C_1} + L\left(\frac{d^2 q_1}{dt^2} - \frac{d^2 q_2}{dt^2}\right) &= \varepsilon(t), \\ L\left(\frac{d^2 q_2}{dt^2} - \frac{d^2 q_1}{dt^2}\right) + \frac{q_2}{C_2} &= 0 \end{aligned} \tag{1}$$

where $q_1(t)$ and $q_2(t)$ are the charges of the two circuits, the C_i ($i = 1, 2$) stand for the respective capacitances of each component circuit, L_1 stands for the inductance of the first return circuit, and L is the coupling inductance between the two circuits. Equations (1) become of simple Hamilton form when $\varepsilon(t) = 0$,

$$H = \frac{p_1^2}{2L_1} + \frac{q_1^2}{2C_1} + \frac{1}{2} L\left(\frac{p_1}{L_1} - \frac{p_2}{L_2}\right)^2 + \frac{q_2^2}{2C_2} \tag{2}$$

where $p_i = L_i\, dq_i/dt$ ($i = 1, 2$) is the conjugate variable of q_i. According to the standard quantization principle, a pair of conjugate observable quantities q_i and p_i are associated with a pair of linear Hermitian operators $\hat{q}_i$ and $\hat{p}_i$, and they satisfy the commutation relation $[\hat{q}_i, \hat{p}_i] = i\hbar$. Thus, the nondissipative inductance coupling circuit is quantized. If we use the following transformations for the charge and current in the quantized Hamiltonian,

$$\begin{aligned} q_1' &= \left(\frac{C_2}{C_1}\right)^{1/4} q_1 \cos\frac{\varphi}{2} - \left(\frac{C_1}{C_2}\right)^{1/4} q_2 \sin\frac{\varphi}{2}, \\ q_2' &= \left(\frac{C_2}{C_1}\right)^{1/4} q_1 \sin\frac{\varphi}{2} + \left(\frac{C_1}{C_2}\right)^{1/4} q_2 \cos\frac{\varphi}{2} \\ p_1' &= \left(\frac{C_1}{C_2}\right)^{1/4} p_1 \cos\frac{\varphi}{2} - \left(\frac{C_2}{C_1}\right)^{1/4} p_2 \sin\frac{\varphi}{2}, \end{aligned} \tag{3}$$

$$p_2' = \left(\frac{C_1}{C_2}\right)^{1/4} p_1 \sin\frac{\varphi}{2} + \left(\frac{C_2}{C_1}\right)^{1/4} p_2 \cos\frac{\varphi}{2} \tag{4}$$

and take

$$\varphi = \operatorname{arctg}\{2[\sqrt{C_2/C_1}(1 + L/L_1) - L_1/L\sqrt{C_1/C_2}]^{-1}\} \tag{5}$$

then the quantized Hamiltonian of the system can be written as

$$\hat{H} = \frac{p_1'^2}{2\mu_1} + \frac{p_2'^2}{2\mu_2} + \frac{1}{2\sqrt{C_1C_2}}(q_1'^2 + q_2'^2) \tag{6}$$

where

$$\frac{1}{\mu_1} = \frac{1}{L_1}\sqrt{\frac{C_2}{C_1}}\left(1 + \frac{L}{L_1}\right)\cos^2\frac{\varphi}{2} + \frac{1}{L}\sqrt{\frac{C_1}{C_2}}\sin^2\frac{\varphi}{2} + \frac{L}{L_1}\sin\varphi \tag{7}$$

$$\frac{1}{\mu_2} = \frac{1}{L_1}\sqrt{\frac{C_2}{C_1}}\left(1 + \frac{L}{L_1}\right)\sin^2\frac{\varphi}{2} + \frac{1}{L}\sqrt{\frac{C_1}{C_2}}\cos^2\frac{\varphi}{2} - \frac{L}{L_1}\sin\varphi \tag{8}$$

Equation (6) is the sum of the Hamiltonians of two independent quantum harmonic oscillators whose frequencies are

$$\omega_1 = \left(\frac{1}{\mu_1\sqrt{C_1C_2}}\right)^{1/2}, \qquad \omega_2 = \left(\frac{1}{\mu_2\sqrt{C_1C_2}}\right)^{1/2} \tag{9}$$

respectively. We thus get the energies and eigenvectors of the system

$$E_{n_1,n_2} = (n_1 + 1/2)\hbar\omega_1 + (n_2 + 1/2)\hbar\omega_2 \qquad (n_1, n_2 = 0, 1, 2, \ldots) \tag{10}$$

$$|\psi_{n_1,n_2}\rangle = |n_1\rangle \otimes |n_2\rangle \qquad (n_1, n_2 = 0, 1, 2, \ldots) \tag{11}$$

where $|n_1\rangle$ and $|n_2\rangle$ represent the eigenvectors of each oscillator with frequencies ω_1 and φ_2, respectively.

3. QUANTUM FLUCTUATIONS OF THE MESOSCOPIC INDUCTANCE COUPLING CIRCUIT IN THE EIGENSTATES OF THE SYSTEM

In order to seek the quantum fluctuations of charge and current in whichever eigenstate of Eq. (11), we introduce the creation and annihilation operators for the two independent oscillators

$$a_i = \left(\frac{\mu_i\omega_i}{2\hbar}\right)^{1/2}\left(q_i{}' + \frac{i}{\mu_i\omega_i}p_i{}'\right),$$

$$a_i^{+} = \left(\frac{\mu_i\omega_i}{2\hbar}\right)^{1/2}\left(q_i{}' - \frac{i}{\mu_i\omega_i}p_i{}'\right) \qquad (i = 1, 2) \tag{12}$$

Then we get

$$q_i' = \left(\frac{\hbar}{2\mu_i\omega_i}\right)^{1/2}(a_i^{+} + a_i), \qquad p_i' = i\left(\frac{\hbar\mu_i\omega_i}{2}\right)^{1/2}(a_i^{+} - a_i) \qquad (i = 1, 2) \tag{13}$$

Therefore, the mean values and mean-square values of q_i' and p_i' $(i = 1, 2)$ in the states given by Eq. (11) can be obtained as, respectively,

$$\langle q_i'\rangle = \langle p_i'\rangle = 0 \qquad (i = 1, 2) \tag{14}$$

$$\langle q_i'^2\rangle = \frac{\hbar}{2}\frac{1}{\mu_i\omega_i}(2n_i + 1) \qquad (i = 1, 2) \tag{15}$$

$$\langle p_i'^2\rangle = \frac{\hbar}{2}\mu_i\omega_i(2n_i + 1) \qquad (i = 1, 2) \tag{16}$$

Then, using Eqs. (3) and (4), we get the mean values and mean-square values of the charge and current of the inductance coupling circuits in the states of (11). They are

$$\langle q_i\rangle = \langle p_i\rangle = 0 \qquad (i = 1, 2) \tag{17}$$

$$\langle q_1^2\rangle = \frac{\hbar}{2}\left(\frac{C_1}{C_2}\right)^{1/2}\left[\frac{2n_1 + 1}{\mu_1\omega_1}\cos^2\frac{\varphi}{2} + \frac{2n_2 + 1}{\mu_2\omega_2}\sin^2\frac{\varphi}{2}\right] \tag{18}$$

$$\langle q_2^2\rangle = \frac{\hbar}{2}\left(\frac{C_2}{C_1}\right)^{1/2}\left[\frac{2n_1 + 1}{\mu_1\omega_1}\sin^2\frac{\varphi}{2} + \frac{2n_2 + 1}{\mu_2\omega_2}\cos^2\frac{\varphi}{2}\right] \tag{19}$$

$$\langle p_1^2\rangle = \frac{\hbar}{2}\left(\frac{C_2}{C_1}\right)^{1/2}\left[\mu_1\omega_1(2n_1 + 1)\cos^2\frac{\varphi}{2} + \mu_2\omega_2(2n_2 + 1)\sin^2\frac{\varphi}{2}\right] \tag{20}$$

$$\langle p_2^2\rangle = \frac{\hbar}{2}\left(\frac{C_1}{C_2}\right)^{1/2}\left[\mu_1\omega_1(2n_1 + 1)\sin^2\frac{\varphi}{2} + \mu_2\omega_2(2n_2 + 1)\cos^2\frac{\varphi}{2}\right] \tag{21}$$

respectively, where the values of μ_i, ω_i $(i = 1, 2)$, and φ are given by Eqs. (7)–(9) and (5). It is evident from the results that, in quantized inductance coupling circuits without power, the mean values of the charge and current are zero in all the eigenstates, while their mean-square values are not zero, i.e., there exist fluctuations of charge and current in the circuits. From Eqs.

(17)–(21), (7)–(9), and (5) we conclude that the magnitudes of the fluctuations of charge and current in each circuit are not only determined by the properties of the circuit itself and the state that the system is in, but are also related to the other circuit and the coupling inductance between them. This signifies that the quantum fluctuations of the charge and current of the two component circuits are connected.

4. QUANTUM FLUCTUATIONS OF THE MESOSCOPIC INDUCTANCE COUPLING CIRCUIT IN THE SQUEEZED VACUUM STATE

Now we study the quantum fluctuations of the charge and current in the squeezed vacuum state. When the circuit has no power, it is assumed to be in the squeezed vacuum state, which takes the form [9]

$$|0,0\rangle_{g_1,g_2} = |0\rangle_{g_1} \otimes |0\rangle_{g_2} = \mathrm{sech}^{1/2}(r_1) \sum_{n=0}^{\infty} \frac{(-e^{i\theta_1} thr_1)^n [(2n)!]^{1/2}}{n! 2^n} |2n\rangle_1 \otimes \mathrm{sech}^{1/2}(r_2) \sum_{m=0}^{\infty} \frac{(-e^{i\theta_2} thr_2)^m [(2m)!]^{1/2}}{m! 2^m} |2m\rangle_2 \tag{22}$$

where r_i and θ_i ($i = 1, 2$) stand for the modulus and argument of the squeezing parameters, respectively. Starting from Eq. (13), the mean values and mean-square values of q_i' and p_i' ($i = 1, 2$) in the squeezed vacuum state given by Eq. (22) can be obtained,

$$\langle q_i' \rangle = \langle p_i' \rangle = 0 \tag{23}$$

$$\langle q_i'^2 \rangle = \frac{\hbar}{2} \frac{\mathrm{sech}^{1/2} r_i}{\mu_i \omega_i} \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n + 2(2n+1)\cos\theta_i \, thr_i] th^{2n} r_i}{(n!)^2 \, 2^{2n}} \tag{24}$$

$$\langle p_i'^2 \rangle = \frac{\hbar}{2} \mu_i \omega_i \, \mathrm{sech}^{1/2} r_i \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n - 2(2n+1)\cos\theta_i \, thr_i] th^{2n} r_i}{(n!)^2 \, 2^{2n}} \tag{25}$$

Then using Eqs. (3) and (4), we get, through careful calculation, the mean values and mean-square values of the charge and current of each circuit as follows:

$$\langle q_i \rangle = \langle p_i \rangle = 0 \qquad (i = 1, 2) \tag{26}$$

$$\langle q_1^2 \rangle = \frac{\hbar}{2} \left(\frac{C_1}{C_2} \right)^{1/2} \left\{ \frac{\mathrm{sech}(r_1)}{\mu_1 \omega_1} \cos^2 \frac{\varphi}{2} \right.$$

$$\times \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n + 2(2n + 1)\cos\theta_1\, thr_1]th^{2n}r_1}{(n!)^2\, 2^{2n}}$$

$$+ \frac{\mathrm{sech}(r_2)}{\mu_2\omega_2} \sin^2\frac{\varphi}{2} \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n + 2(2n + 1)\cos\theta_2\, thr_2]th^{2n}r_2}{(n!)^2\, 2^{2n}}\Bigg\} \tag{27}$$

$$\langle q_2^2\rangle = \frac{\hbar}{2}\left(\frac{C_2}{C_1}\right)^{1/2}\Bigg\{\frac{\mathrm{sech}(r_1)}{\mu_1\omega_1}\sin^2\frac{\varphi}{2}$$

$$\times \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n + 2(2n + 1)\cos\theta_1\, thr_1]th^{2n}r_1}{(n!)^2\, 2^{2n}}$$

$$+ \frac{\mathrm{sech}(r_2)}{\mu_2\omega_2} \cos^2\frac{\varphi}{2} \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n + 2(2n + 1)\cos\theta_2\, thr_2]th^{2n}r_2}{(n!)^2\, 2^{2n}}\Bigg\} \tag{28}$$

$$\langle p_1^2\rangle = \frac{\hbar}{2}\left(\frac{C_2}{C_1}\right)^{1/2}\Bigg\{\mu_1\omega_1\,\mathrm{sech}(r_1)\cos^2\frac{\varphi}{2}$$

$$\times \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n - 2(2n + 1)\cos\theta_1\, thr_1]th^{2n}r_1}{(n!)^2\, 2^{2n}}$$

$$+ \mu_2\omega_2\,\mathrm{sech}(r_2)\sin^2\frac{\varphi}{2}$$

$$\times \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n - 2(2n + 1)\cos\theta_2\, thr_2]th^{2n}r_2}{(n!)^2\, 2^{2n}}\Bigg\} \tag{29}$$

$$\langle p_1^2\rangle = \frac{\hbar}{2}\left(\frac{C_1}{C_2}\right)^{1/2}\Bigg\{\mu_1\omega_1\,\mathrm{sech}(r_1)\sin^2\frac{\varphi}{2}$$

$$\times \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n - 2(2n + 1)\cos\theta_1\, thr_1]th^{2n}r_1}{(n!)^2\, 2^{2n}}$$

$$+ \mu_2\omega_2\,\mathrm{sech}(r_2)\cos^2\frac{\varphi}{2}$$

$$\times \sum_{n=0}^{\infty} \frac{(2n)![1 + 4n - 2(2n + 1)\cos\theta_2\, thr_2]th^{2n}r_2}{(n!)^2\, 2^{2n}}\Bigg\} \tag{30}$$

where the values of μ_i, ω_i ($i = 1, 2$), and φ are given by Eqs. (7)–(9) and (5). From the results we find that, in the quantized inductance coupling circuits, the mean values for both charge and current in each circuit are zero in the squeezed vacuum state, but their mean-square values are not zero, analogous to the situation in the ordinary vacuum state. This indicates that there exist quantum fluctuations for both charge and current in each circuit, and that not only is the magnitude of the fluctuation related to the device and the state that the system is in, but it is also associated with the other circuit and the magnitude of the coupling inductance. Therefore, the quantum fluctuations of the charge and current in each component circuit are connected when the system is in the squeezed vacuum state.

5. CONCLUSIONS

In conclusion, for nondissipative mesoscopic inductance coupling circuits, there exist quantum fluctuations of the charge and current either in the eigenstates of the system or in the squeezed vacuum states, and the fluctuations in each component circuit are connected. We believe that this quantum effect generally exists in other mesoscopic circuits and is worth further investigation. The study of this effect is significant for the design of miniature circuits and the reduction of quantum noise.

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